Graph Theory. Third Edition. *By Reinhard Diestel.* Springer-Verlag, Berlin, 2005. $49.95. xvi+411 pp., softcover. ISBN 3-540-26183-4.

Graphs seem to be everywhere nowadays, from the Internet and its power law graphs to terrorists' networks and gene networks. They are also making mathematical news. The perfect graph conjecture was recently solved. Also, we have the completion of Robertson and Seymour's heroic series on graph minors with its many applications. Even the recent "back to the axioms" proof of the Jordan curve theorem rests on the fact that the complete bipartite graph on two sets of three vertices in not embeddable in the plane. In our experience, most mathematicians (applied or not) don't know the first thing about graph theory. They glaze over at the new developments and have to duck when simple combinatorial problems cross their paths. If you want to change, have we got a book for you!

Reinhard Diestel has written a deep, clear, wonderful book about graph *theory*. It is filled with examples from the heart of the subject: matching theory, connectivity, planarity, coloring, flows, Ramsey theory, and random graphs all are developed in separate chapters. To paraphrase from the introduction (page viii):

> A typical chapter begins with a brief discussion of what are the guiding questions in the area it covers, continues with a succinct account of its classical results (often with simplified proofs) and then presents one or two deeper theorems that bring out the full flavor of that area. The proofs of these latter results are typically preceded by an informal account of their main ideas. They are then presented formally.

The writing is terse but clear. You can really follow the proofs without a graph theorist's intuition.

A highlight of the book is the only accessible account of what has come to be called Robertson–Seymour theory. To introduce this theory, consider the following ways of reducing a graph G (which we assume to be undirected, with loops and parallel edges allowed): (i) delete an edge; (ii) contract an edge; (iii) delete an isolated vertex. (Contracting an edge $e = uv$ means removing u and v and adding a new vertex v_e which becomes adjacent to all the former neighbors of u and v.) Any graph G′ which can be produced by successive applications of these operations is called a *minor* of G. Every graph that is isomorphic to a minor of G is also called a minor of G.

Now consider the two graphs in Figure 1. The left-hand graph is K(5), the complete graph on 5 vertices; the right-hand graph is K(3,3), the complete bipartite graph on 2 sets of 3 vertices. A classical theorem of Kuratowski is equivalent to the statement that a graph can be drawn in the plane without crossing edges if and only if it does not have K(5) or K(3,3) as a minor. One might naturally ask whether a similar result exists for graphs which are embeddable in higher genus surfaces. In other words, can such graphs be characterized by excluding some finite list of graphs as forbidden mi-

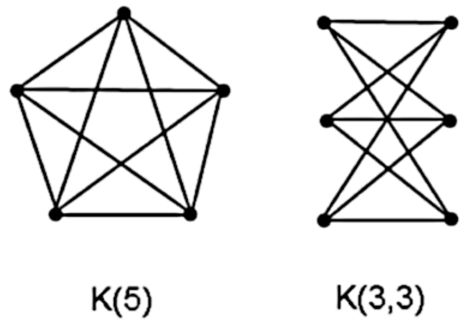

Fig. 1 *Two nonplanar graphs.*

nors? It turns out that not only is this true, but in fact something much stronger holds. Let us call a graph property *hereditary* if it is "closed under taking minors." This just means that if a graph has some property, then any minor of the graph also has that property. Thus, being embeddable in a fixed surface is a hereditary property. The Robertson–Seymour theorem says that any hereditary graph property can be characterized by a finite list of excluded minors. The proof of this result has taken over twenty years and runs over some twenty papers. Diestel's book contains an overview, many applications, and real proofs of essential parts. This work also has profound applications to graph algorithms. Let us explain. A nonmathematical reader may say, "That's nice. You can work hard and wind up proving you can draw a picture of a graph in the plane. So what?" If a graph is planar, many algorithmic tasks (such as deciding colorability, which is used in scheduling problems and many others) can be done efficiently. These tasks require exponential running times for general graphs. Similarly, for graphs characterized by a finite list of excluded minors, many algorithmic tasks can be accomplished by cubic time algorithms (or better). The methods underlying the proof, for example, tree width (a kind of dimension theory for general graphs), seem destined to play fundamental roles in areas such as phylogeny.

Diestel's book is written with remarkable expository care. For example, the margins are filled with pointers to where the present lemma is used and where a needed lemma can be found. The margins highlight where on the page a particular definition can be found.

The book is designed as a textbook for an upper division undergraduate or beginning graduate course. It contains a wealth of exercises and a section of hints. We have also found it very useful for self study; the writing and cross-referencing are clear enough that you can dip into a section or individual result without having to start from the beginning. Watch out, though, since the writing (and the author's infectious enthusiasm) can keep you reading long after you have found what you first wanted! The notes at the end of each chapter give brief but good historical summaries, pointers to specialist literature, open problems, and philosophy.

This is a serious book about the heart of graph theory. It does not have a collection of made-up fake applications. The book has depth and integrity. The "right" versions of theorems are proved honestly without handwaving, yet we don't find much straying toward esoteric corners. The third edition has incorporated a number of recent developments. Of course, a 400-page book cannot cover everything. The author laments two omissions: algebraic graph theory and applications. We can recommend the treatment of these in *Algebraic Graph Theory*, (Cambridge University Press, 1993) by Norman Biggs, and *Spectral Graph Theory* by Fan Chung (AMS, 1997).

If you want to meet graph theory in its grown-up modern form, get a copy of this book and teach a course with it. You will be well rewarded.

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