Graph theory (2nd edn.), by Reinhard Diestel. Pp. 312. £24. 2000. ISBN 0 387 98976 5 (Springer-Verlag).

This is the second edition of a book which first appeared in 1997 and which does not seem to have been reviewed in the *Gazette*. It has already established itself as an excellent 'modern' account of graph theory, readable and yet both rigorous and scholarly. The level of sophistication makes it ideal for a postgraduate course; it would, I think, be beyond the level of most undergraduate courses in the UK.

Noting that almost two decades had passed 'since the appearance of these graph theory texts that still set the agenda for most introductory courses taught today', the author asks in his introduction 'what are, today, the essential areas, methods and results that should form the centre of an introductory graph theory course aiming to equip its audience for the most likely developments ahead?' The answer presumably is found in the choice of chapter topics : matchings; connectivity; planar graphs; colouring; flows; substructures in dense graphs and in sparse graphs; Ramsey theory; Hamiltonian cycles; random graphs; minors, trees and well-quasi-orderings.

Each chapter ends with scholarly notes of a bibliographical and historical nature. These give a feel for how the subject is developing. Each chapter also has a set of exercises and, as a bonus, the second edition includes hints for all of these exercises. This must make the book more attractive as a course text.

The author has thought very carefully about how to present his material. Topics are dealt with thoroughly; for example, Kuratowski's theorem on planar graphs is carefully proved, and other characterisations of planar graphs such as those in terms of cycle spaces (MacLane, 1937) and in terms of abstract duals (Whitney, 1933) are presented. Some standard theorems are presented with more than one proof. For example, Menger's theorem (1927), which asserts that the minimum number of vertices separating vertices A and B is equal to the maximum number of disjoint paths from A to B, has three different proofs, the third being an ingenious and previously unpublished proof due to Böhme, Göring and Harant. As another example, I mention the Perfect Graph Theorem of Lovász (1972). A subgraph G' of a graph G is induced if it is obtained from G by throwing away some of the vertices of G and all the edges of G incident with these vertices. A graph is perfect if, for every induced subgraph H, the chromatic number of H is equal to the clique number of H, i.e. the largest value of r such that H contains the complete graph on r vertices as a subgraph. Lovász's result is that a graph is perfect if and only if its complement is perfect. Diestel gives Lovász's original proof 'which is still unsurpassed in its clarity and the amount of "feel" for the problem it conveys', as well as a very recent short linear algebra proof due to Gasparian (1996).

The biggest changes in the second edition occur in the final chapter on minors. A graph X is a minor of Y if Y contains as a subgraph a graph G which can be obtained from X by a sequence of edge contractions. The Graph Minor Theorem 'which dwarfs any other result in graph theory and may doubtless be counted among the deepest theorems that mathematics has to offer' asserts that in every infinite set of finite graphs there must be two such that one is a minor of the other. A sophisticated way of saying this is that the finite graphs are well-quasi-ordered by the minor relation. Robertson and Seymour proved this result via a series of papers totalling over 500 pages, so a full proof here is out of the question. Instead, an outline of the proof is presented. However, in the second edition, Diestel presents full new proofs of two of the major Robertson- Seymour results, including the one which asserts that excluding a graph H as a minor bounds the tree-width if and only if the graph H is planar.

The book has its own web site

http://www.math.uni-hamburg.de/home/diestel/books/graph.theory

where further information about the book can be obtained. Undoubtedly, this is a book which will be a standard graduate text for many years.

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