About the sixth edition

This sixth edition offers a major update on previously existing material, while also adding some new sections on topics that reflect the directions which graph theory has taken more recently.

The largest addition of new material comes in Chapter 7, *Extremal graph theory*, where I have modernized the treatment of the regularity lemma by adding a section on 'regularity tools': the counting lemma, and the removal lemma. This is followed by a short new section on Szemerédi's theorem. Its special case, Roth's theorem, is proved there as an application of those regularity tools. The proof of the regularity lemma itself, and the introductory section on how it is typically applied, have been revised but essentially remained as before.

There is an entirely new section on χ-boundedness in Chapter 5. Gallai's A-paths theorem now shows up explicitly in Chapter 3, as does Nash-Williams's cycle-decomposition theorem in Chapter 8, a true classic in infinite graph theory. It is proved from Laviolette's theorem, which is also newly included.

Some classical theorems in the book have been furnished with shorter and more intuitive proofs, as these were recently discovered or brought to my attention. They include Lovász's perfect graph theorem in Chapter 5 and Seymour's six-flow theorem in Chapter 6. For several others I streamlined the existing proofs, sometimes considerably, as I revisited them myself for teaching. These include Tutte's flow polynomial theorem in Chapter 6, Turán's theorem in Chapter 7, and the Erdős-Chvátal theorem in Chapter 10. Even the five-colour theorem got a sweet and very short additional proof, which I had not previously known or thought of. Finally, the tree-of-tangles theorem in Chapter 12 has two even more powerful proofs now, neither of which existed when I wrote the section on tangles for the 5th edition.

However I have resisted including new proofs of classical results that are shorter but *not* more intuitive. The main reason for including any proofs at all remains to elucidate how graph theory typically works: with an emphasis on its diversity of methods, but also on single recurrent ideas which the reader may collect in their own personal toolkit. This is also the reason why, after many editions in which I included it, I have now omitted one of the two original proofs of the induced Ramsey theorem. The exposition now concentrates on the amalgamation proof, which turned out to be more fertile as Ramsey theory developed.

But I have added some further post-mortem analyses of technical proofs, to elucidate their main ideas with the benefit of hindsight. This applies in particular to the new material in Chapter 7, where an informal discussion precedes the counting lemma and some extensive discussion follows the removal lemma. This includes some magic insights pointed out to me by Mathias Schacht, on whose notes all this material is based.

My thanks also go to Nicolas Trotignon for his advice on how to prove the perfect graph theorem in a way that carries a maximum of insight, to Alex Scott for his advice on the new section on χ-boundedness, and to Qi Renrui for pointing out a shortcut in proof of Lemma 8.4.6. And, as with every new edition, to the many readers that generously pointed out potential improvements: they were as highly appreciated as ever!

Finally, let me take this opportunity to point to another book which I have just completed: one on applying tangles in the real world. The graph tangles introduced in Chapter 12.5 were the starting point for a development which, over the past 10 years, has led to an axiomatic theory of 'abstract' tangles. These, precisely because they are so general, appear to be much more widely applicable than graph tangles, although their theory is no more difficult. Readers of Chapter 12.5 will be over-equipped to read that book if they feel drawn to applying tangles. See

`https://tangles-book.com`

for more on the book and its related open-source software.

May 2024 *RD*