

About the third edition

There is no denying that this book has grown. Is it still as ‘lean and concentrating on the essential’ as I said it should be when I wrote the preface to the first edition, now almost eight years ago?

I believe that it is, perhaps now more than ever. So why the increase in volume? Part of the answer is that I have continued to pursue the original dual aim of offering two different things between one pair of covers:

- a reliable first introduction to graph theory that can be used either for personal study or as a course text;
- a graduate text that offers some depth in selected areas.

For each of these aims, some material has been added. Some of this covers new topics, which can be included or skipped as desired. An example at the introductory level is the new section on packing and covering with the Erdős-Pósa theorem, or the inclusion of the stable marriage theorem in the matching chapter. An example at the graduate level is the Robertson-Seymour structure theorem for graphs without a given minor: a result that takes a few lines to state, but one which is increasingly relied on in the literature, so that an easily accessible reference seems desirable. Another addition, also in the chapter on graph minors, is a new proof of the ‘Kuratowski theorem for higher surfaces’—a proof which illustrates the interplay between graph minor theory and surface topology better than was previously possible. The proof is complemented by an appendix on surfaces, which supplies the required background and also sheds some more light on the proof of the graph minor theorem.

Changes that affect previously existing material are rare, except for countless local improvements intended to consolidate and polish rather than change. I am aware that, as this book is increasingly adopted as a course text, there is a certain desire for stability. Many of these local improvements are the result of generous feedback I got from colleagues using the book in this way, and I am very grateful for their help and advice.

There are also some local additions. Most of these developed from my own notes, pencilled in the margin as I prepared to teach from the book. They typically complement an important but technical proof, when I felt that its essential ideas might get overlooked in the formal write-up. For example, the proof of the Erdős-Stone theorem now has an informal post-mortem that looks at how exactly the regularity lemma comes to be applied in it. Unlike the formal proof, the discussion starts out from the main idea, and finally arrives at how the parameters to be declared at the start of the formal proof must be specified. Similarly, there is now a discussion pointing to some ideas in the proof of the perfect graph theorem. However, in all these cases the formal proofs have been left essentially untouched.

The only substantial change to existing material is that the old Theorem 8.1.1 (that cr^2n edges force a TK^r) seems to have lost its nice (and long) proof. Previously, this proof had served as a welcome opportunity to explain some methods in sparse extremal graph theory. These methods have migrated to the connectivity chapter, where they now live under the roof of the new proof by Thomas and Wollan that $8kn$ edges make a $2k$ -connected graph k -linked. So they are still there, leaner than ever before, and just presenting themselves under a new guise. As a consequence of this change, the two earlier chapters on dense and sparse extremal graph theory could be reunited, to form a new chapter appropriately named as *Extremal Graph Theory*.

Finally, there is an entirely new chapter, on infinite graphs. When graph theory first emerged as a mathematical discipline, finite and infinite graphs were usually treated on a par. This has changed in recent years, which I see as a regrettable loss: infinite graphs continue to provide a natural and frequently used bridge to other fields of mathematics, and they hold some special fascination of their own. One aspect of this is that proofs often have to be more constructive and algorithmic in nature than their finite counterparts. The infinite version of Menger's theorem in Section 8.4 is a typical example: it offers algorithmic insights into connectivity problems in networks that are invisible to the slick inductive proofs of the finite theorem given in Chapter 3.3.

Once more, my thanks go to all the readers and colleagues whose comments helped to improve the book. I am particularly grateful to Imre Leader for his judicious comments on the whole of the infinite chapter; to my graph theory seminar, in particular to Lilian Matthiesen and Philipp Sprüssel, for giving the chapter a test run and solving all its exercises (of which eighty survived their scrutiny); to Angelos Georgakopoulos for much proofreading elsewhere; to Melanie Win Myint for recompiling the index and extending it substantially; and to Tim Stelldinger for nursing the whale on page 366 until it was strong enough to carry its baby dinosaur.